



QUANTITATIVE METHODS FOR MANAGERS

September 2021

Time allowed

Three hours

Instructions

- Write the question number next to each answer in your answer booklet.
- You are not required to rewrite the question in your answer booklet.
- Ensure that you pay particular attention to words in **bold**.
- Show all your workings.

Information

- Different questions may carry a different number of marks.
- Marks for each question are shown in [].
- Questions start on page 8.

Advice

- Read each question carefully before you start to answer it.
- Use the full time permitted and check all your answers.

Materials

- Notes or books are not permitted.
- Non-programmable calculators are permitted.



ICM

STATISTICAL FORMULAE

Frequency Distributions

Arithmetic Mean

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Weighted average

$$\text{Weighted average} = \frac{\sum xw}{\sum w}$$

Range

Range = Highest value – Lowest value

Quartile deviation

$$\text{Quartile deviation} = \frac{\text{Upper quartile} - \text{Lower quartile}}{2}$$

Standard deviation

$$S.D. = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{\sum f_i}}$$

Variance

$$\text{Variance} = (S.D.)^2$$

Coefficient of variation

$$\text{Coefficient of variation} = \frac{\text{S.Deviation}}{\text{Mean}} \times 100$$

Pearson coefficient of skewness (Sk)

$$Sk = \frac{3(\text{Mean} - \text{Median})}{S.D.}$$

Standard Errors and Confidence Intervals

Standard normal deviation

$$z = \frac{x - \mu}{\sigma}$$

Standard error of the mean

$$S.E. = \frac{s}{\sqrt{n}}$$

Standard error of a proportion

$$S.E. = \sqrt{\frac{pq}{n}}$$

where p = sample proportion, and $q = 1 - p$.

Confidence Interval for population mean (μ)

$$\bar{X} \pm Z_{\alpha} \times S.E.$$

Confidence Interval for population proportion (π)

$$p \pm Z_{\alpha} \times S.E.$$

Regression/Correlation

Regression lines

Regression line of y on x

Line equation: $y = a + bx$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \quad a = \frac{\sum y - b\sum x}{n}$$

Correlation Coefficient (r)

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

Rank Correlation Coefficient

$$\text{Spearman's coefficient of rank correlation} = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where n = the number of pairs, and d = the difference between ranking of the same item in each series.

Index Numbers

Laspeyres price index

$$\text{Index} = \frac{\sum(p_n \cdot q_o)}{\sum(p_o \cdot q_o)} \times 100$$

Paasche price index

$$\text{Index} = \frac{\sum(p_n \cdot q_n)}{\sum(p_o \cdot q_n)} \times 100$$

Price relative

$$\text{Price relative} = \frac{P_n}{P_o} \times 100$$

Base changing

$$\text{New index number} = \frac{\text{Old index number}}{\text{Old index number of new base period}} \times 100$$

Asset revaluation

$$\text{New valuation} = \text{Original value} \times \frac{\text{New price index}}{\text{Original price index}}$$

ANSWER ANY FIVE QUESTIONS FROM THE FOLLOWING EIGHT QUESTIONS

1. (a) $\frac{5}{6}$ of employees at a small company received a bonus this month.
State how many people the company employ in total if 32 people received a bonus. [3 marks]
- (b) Simplify each of the following fully:
- (i) $8z^2 \times \frac{1}{4} z^9$ [2 marks]
- (ii) $117a^5b \div 9a^2b^6$ [2 marks]
- (iii) $(4u-2)/(24u^2-20u+4)$ [4 marks]
- (c) Solve the simultaneous equations. Give your answers as fractions in their simplest form:
- $$12c = 34 + 9d$$
- $$2d - 3c - \frac{1}{2} = 0$$
- [9 marks]

2. **Total revenue and total cost in GBP (£) for a company:**

$$TR = 40x - 8x^2$$

$$TC = 8 + 16x - x^2$$

Where:

TR = Total revenue (£'000)

TC = Total cost (£'000)

x = Output ('000 units)

Note:

The company only outputs a whole number of units

- (a) Calculate the level of output at which total profit is maximised. [8 marks]
- (b) Calculate the price per unit that will be charged at this output to the nearest pence. [7 marks]
- (c) Calculate the profit at this output to the nearest GBP (£). [5 marks]
3. The following table gives information about the weights of students in a year group of 200:

Student weights

Weight (w kg)	Frequency
$40 \leq w \leq 50$	5
$50 < w \leq 60$	21
$60 < w \leq 70$	89
$70 < w \leq 80$	68
$80 < w \leq 90$	10
$90 < w \leq 100$	7

- (a) Plot an ogive (cumulative frequency curve) to represent this data. [12 marks]
- (b) Give an estimate for the value of the interquartile range based on the ogive you plotted in part (a). [8 marks]

4. The manager of a garden centre is investigating the relationship between the volume of fertiliser used on plants ($x \text{ cm}^3$) and the maximum height that the plants grow to ($y \text{ cm}$).

x	10	20	30	40	50	60	70
y	34	56	70	77	79	76	72

- (a) Calculate the Pearson's coefficient of correlation, R . Give your answer correct to **three** decimal places. [11 marks]
- (b) State the effect on the value of R if the maximum height were recorded in m instead of cm. [1 mark]
- (c) Calculate the equation of the regression line (of y on x). Give all values in your equation correct to **two** significant figures. [8 marks]

5. (a) A company produces three items of oak garden furniture: benches, loungers and chairs.

Output and prices are shown in the following table:

	2018		2019		2020	
	Q_0	P_0	Q_1	P_1	Q_2	P_2
Benches	213	45	304	60	304	79
Loungers	98	82	191	113	278	115
Chairs	302	94	424	123	535	151

Calculate the following price indices for each of the three years, using 2018 as the base year, giving your answer correct to **one** decimal place:

- (i) Laspeyres [6 marks]
- (ii) Paasche [6 marks]

- (b) The following information on the Retail Price Index is taken from the August 2020 Labour Market Trends:

August 2020 Labour Market Trends

	Food	Catering	Alcohol	Tobacco	Housing	Fuel & Light	Household Goods
Weight	136	49	80	34	186	41	72
Index Jan. 2020	141.0	179.2	171.1	200.1	172.1	133.2	135.6
Index July 2020	142.2	182.7	175.0	205.2	180.9	131.2	137.3
	Household Services	Clothing & Footwear	Personal Goods	Motoring	Fares & Other Travel	Leisure Goods	Leisure Services
Weight	52	56	40	128	20	47	59
Index Jan. 2020	142.7	116.3	166.7	162.9	166.6	123.7	177.8
Index July 2020	143.8	115.9	169.8	165.9	170.9	123.9	182.5

Calculate the index as of July 2020 if the index stood at 154.4 in January 2020. Give your answer correct to **one** decimal place.

[8 marks]

6. The British and Irish Lions rugby squad contained 50 players. The nationalities and playing positions of these players are shown in the following table:

		Nationality			
		English	Welsh	Scottish	Irish
Playing Position	Forward	14	5	2	6
	Back	8	7	2	6

- (a) (i) Explain what is meant by continuous data. [2 marks]
(ii) Explain what is meant by discrete data. [2 marks]
(iii) State whether the amount of Welsh backs in the rugby squad is an example of continuous or discrete data. [1 marks]
- (b) A player was selected at random from the squad for a radio interview. State as a fraction in its simplest form the probability that the player was:
- (i) a Welsh back [1 mark]
(ii) English [2 marks]
(iii) not English [2 marks]
(iv) Irish, given that the player was a back [2 marks]
(v) a forward, given that the player was not Scottish [2 marks]
- (c) Four players were selected at random from the squad to visit a school. Calculate the probability that all four players were English. Give your answer as a fraction in its simplest form. [6 marks]
7. Kate listens to music for Y minutes each day. Y is a random variable which may be modelled by a normal distribution with a mean of 55 minutes and a standard deviation of 23 minutes.
- (a) Calculate the probability correct to **three** significant figures that on a particular day the time Kate spends listening to music is:
- (i) less than 45 minutes [5 marks]
(ii) between 50 and 100 minutes [10 marks]
- (b) Calculate the probability that on 14 randomly selected days Kate spends more than 45 minutes listening to music on exactly eight of those days. Give your answer correct to **three** significant figures. [5 marks]

8. (a) Calculate how much Harry should pay as a lump sum for an annual annuity of £4,000, starting at the end of year 1 and continuing for 14 years in total. The annual rate of discount is 9.87%. [7 marks]
- (b) William can afford to repay \$720 a month for 25 years on his mortgage. Interest is calculated at 24% per annum, payable monthly. Calculate how large a mortgage William can afford. [7 marks]
- (c) The following table shows the expected cash flow (€million) of two investment projects over seven time periods.

Expected cash flow (€million) of two investment projects over seven time periods

	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7
Project A	-35	0	1	6	9	9	12	15
Project B	-7	1	1	2	2	5	0	1

Calculate the average rate of return on Projects A and B per time period. Give both answers as a percentage correct to **two** decimal places. [6 marks]

END OF QUESTIONS